Applications of Computer Algebra – ACA 2025
July 14-18, 2025
Session *"Computer algebra modeling in physics, classical and celestial mechanics, and engineering"*

Classification of Universal Decision Elements Using Computer Algebra Systems

Arkadiusz Orłowski[1] [arkadiusz_orlowski@sggw.edu.pl]

[1] Instytut Informatyki Technicznej, WULS-SGGW, Warszawa, Poland

The concept of Universal Decision Elements (UDEs) emerged in the 1950s [1] and underwent further development in the 1960s-70s [2,3]. These early efforts, although insightful, were constrained by the limitations of contemporary hardware and lacked exhaustive formal analysis. Here, we revisit the problem from a modern perspective, leveraging the power of Computer Algebra Systems to perform a complete and verifiable classification of all possible UDEs within a clearly defined logical and functional framework.

We introduce a rigorous formalization of the UDE concept, define the precise criteria for their universality, and use symbolic computation to systematically analyze the entire space of candidate logical functions. The resulting classification reveals both known and previously unrecognized universal elements, providing a comprehensive map of the UDE landscape.

Finally, we propose a series of generalizations of the classical UDE framework, extending it to accommodate reversible computing and quantum information processing. By reframing UDEs in terms compatible with reversibility, we contribute to the rapidly evolving field of reversible and quantum computation, and suggest that long-overlooked and largely forgotten constructs from mid-20th-century logic design may acquire new relevance in the emerging paradigms of reversible logic synthesis and post-classical computation.

Keywords
Universal Decision Element

References

[1] B. SOBOCIŃSKI, On a universal decision element. *The Journal of Computing Systems* **volume** 1, 71–80 (1953).
[2] E. FOXLEY, Determination of the set of all four-variable formulae corresponding to universal decision elements using a logical computer. *Mathematical Logic Quarterly* **volume** 10, 302–314 (1964).
[3] J. C. MUZIO, Partial universal decision elements. *Notre Dame Journal of Formal Logic* **volume** 15, 133–140 (1974).