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Possible Orderings of Mode, Median, and Mean in Unimodal Distributions

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It is obvious that in symmetric and unimodal probability distribution functions (PDFs), the *mode*, *median*, and *mean* coincide. In asymmetric distributions, the typical ordering of these measures follows a predictable pattern: mode < median < mean (for right-skewed PDFs) or mean < median < mode (for left-skewed PDFs). This pattern is so entrenched that even in modern textbooks it is seldom, if ever, disputed.

This raises a natural and surprisingly deep question: *Are other orderings of mode, median, and mean possible?* In particular, are *all four* remaining orderings realizable? One can show that some such different orderings are indeed possible — e.g., by forming artificial mixtures of PDFs or adding a local peak to distort the density. However, such constructions usually lead to multimodal and highly irregular distributions, undermining their statistical relevance. While scattered examples in the literature [1,2] show that *some* alternative orderings may also arise even in unimodal PDFs, there has been *no systematic study* exploring the full set of six possible orderings under the unimodality constraint.

Here, we undertake such a study, leveraging the power of Computer Algebra Systems (CAS) and symbolic-numeric tools which allow us to explore complex nonlinear relationships between distribution parameters and the resulting positions of the mode, median, and mean. As a result we find examples of PDFs providing all six orderings. We compute and plot the corresponding PDFs to verify unimodality and to visually confirm the relative locations of the parameters. It shows that even basic statistical notions can exhibit unexpected structural richness, and that CASs provide a powerful framework for resolving such challenging questions. Possible applications in physics and engineering are also given.

Keywords
Mode, Median, Mean

References
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