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Convergence order in trajectory estimation with piecewise Bézier cubics based on reduced data

Ryszard Kozera[1,2]***, Magdalena Wilkołazka***[3] [ryszard_kozera@sggw.edu.pl]

[1] Warsaw University of Life Sciences - SGGW, Warsaw, Poland
[2] The University of Western Australia, Perth, Australia
[3] The John Paul II Catholic University of Lublin, Lublin, Poland

We discuss the problem of fitting reduced data $\mathcal{Q}_m = \{q_i\}_{i=1}^m$ in arbitrary Euclidean space $\mathbb{E}^n$. In our setting the interpolation knots $\{t_i\}_{i=0}^m$ (with $q_i = \gamma(t_i)$) are unknown and need to be compensated by certain $\hat{\mathcal{T}} = \{\hat{t}_i\}_{i=0}^m$ (see e.g. [1]). Various fitting schemes combined with some recipes for $\hat{\mathcal{T}}$ were studied e.g. in [1-3] (for dense $\mathcal{Q}_m$) or [4-5] (for sparse $\mathcal{Q}_m$). In case of $\mathcal{Q}_m$ dense, the convergence rate (and its sharpness) for a selected interpolation scheme $\hat{\gamma}$ (based on $\mathcal{Q}_m$ and $\hat{\mathcal{T}}$) in approximating γ is a task to examine - see e.g. [2-4]. We analyze the problem of partially fitting $\mathcal{Q}_m$ by merely interpolating $\hat{\mathcal{Q}}_m = \{q_0, q_3, q_6, \ldots, q_{m=3k}\}$ with piecewise cubic Bézier curve $\hat{\gamma}_B$ (see [6]). The other points serve only as control points. *A sharp quadratic order* in γ estimation by $\hat{\gamma} \circ \phi$ (with $\phi : [0, T] \to [0, \hat{T}]$) is proved. Numerical and symbolic computation in *Mathematica* is used to confirm the latter.

Keywords
Interpolation, Reduced Data, Convergence Orders and Sharpness

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