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# Symbolic computations in studying the stability of nonlinear oscillations of the mathematical pendulum

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The mathematical pendulum is a simple mechanical system with one degree of freedom and its motion is determined by the second order ordinary differential equation [1]. Its general solution may be written in terms of the Jacobi elliptic functions and describes a periodic motion of the pendulum in the domain $\varphi \in [-a, a]$, where $a$ is the amplitude of oscillations. From the other side, using the Poincare-Lindstedt method [2], one can construct this periodic solution in the form of power series in the amplitude $a$ that is assumed to be small [3]. As the oscillation frequency depends on the amplitude, the periodic solution is unstable in Lyapunov sense. The main aim of this talk is to demonstrate the most important and useful algorithms for studying the stability of periodic solutions, considering the nonlinear oscillations of the mathematical pendulum as an example. Implementation of the corresponding algorithms involves quite cumbersome symbolic computation which may be performed efficiently with the aid of the computer algebra systems, for instance, Wolfram Mathematica.

**Keywords**
Nonlinear oscillations, Stability, Symbolic calculation, Wolfram Mathematica

**References**

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