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Resonances and Periodic Motion of Atwood's Machine with Two Oscillating Bodies

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The swinging Atwood machine under consideration consists of two masses m_1, m_2 attached to opposite ends of a massless inextensible thread wound round two massless frictionless pulleys of negligible radius (see [1]). Both masses m_1 and m_2 are allowed to oscillate in a plane. Such a system has three degrees of freedom and its equations of motion may be written in the form

$$\begin{aligned} r\ddot{\varphi} &= -g\sin\varphi - 2\dot{r}\dot{\varphi}, \\ (L-r)\ddot{\psi} &= -g\sin\psi + 2\dot{r}\dot{\psi}, \\ (m_1+m_2)\ddot{r} &= m_1 g\cos\varphi - m_2 g\cos\psi + m_1 r\dot{\varphi}^2 - m_2(L-r)\dot{\psi}^2, \end{aligned} \tag{1}$$

where the dot above the symbol denotes a total time derivative of the corresponding function, the variables r, φ, ψ describe geometrical configuration of the system, g is a gravity constant. Note that equations of motion (1) are essentially nonlinear, and their general solution cannot be found in symbolic form. However, there exist periodic solutions which may be represented in the form of power series (see [2,3]). In the present talk, we construct such periodic solutions and demonstrate that they exist only if the frequencies of the bodies oscillations are commensurable or a resonance of frequencies takes place.

Keywords
Swinging Atwood machine, Equations of motion, Periodic solutions, Resonances

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