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Fitting Sparse Reduced Data

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We discuss the problem of fitting data points $\mathcal{Q}_m = \{q_i\}_{i=0}^m$ in arbitrary Euclidean space $\mathbb{E}^n$. It is additionally assumed here, that the corresponding interpolation knots $\{t_i\}_{i=0}^m$ remain unknown and as such they need to be somehow replaced by $\hat{\mathcal{T}} = \{\hat{t}_i\}_{i=0}^m$ (subject to $\hat{t}_i < \hat{t}_{i+1}$). Here, without loss of generality $\hat{t}_0 = 0$ and $\hat{t}_m = T$, for some $T > 0$. In the case of Q_m dense the issue of convergence rate of a given interpolation scheme $\hat{\gamma}$ (based on $\mathcal{Q}_m$ and $\hat{\mathcal{T}}$) in approximating γ (satisfying $\gamma(t_i) = q_i$) has been extensively studied (see e.g. [1]). In contrast for $\mathcal{Q}_m$ sparse a possible criterion to select the new knots $\hat{\mathcal{T}}$ is to minimize:

$$\mathcal{J}(\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_{m-1}) = \int_0^T \|\ddot{\gamma}_N(\hat{t})\| d\hat{t}, \tag{1}$$

where $\hat{\gamma}_N$ is a natural spline based on $\mathcal{Q}_m = \{q_i\}_{i=0}^m$ and $\hat{\mathcal{T}}$. Finding such optimal knots $\hat{\mathcal{T}}^{opt}$ forms a highly nonlinear optimization task (see e.g. [2]). One of the computational schemes handling (1) (called Leap-Frog) relies on the composition of overlapping univariate optimizations schemes - see [3]. We discuss special conditions under which the unimodality of these univariate functions holds and show the robustness in case of their perturbation.

Keywords
Interpolation, Optimization, Reduced Data

References

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