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On the Equilibria and Bifurcations of a Rotating Double Pendulum

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The double pendulum, a simple system of classical mechanics, is widely studied as an example of, and testbed for, chaotic dynamics. In [1], Maiti et al. study a generalization of the simple double pendulum with equal point-masses at equal lengths, to a rotating double pendulum, fixed to a coordinate system uniformly rotating about the vertical. In this work, we study a further generalization: a rotating double pendulum constructed from *physical* pendula, or rigid 3D bodies. We examine what equilibrium configurations exists for the system across a comparatively large parameter space, as well as what bifurcations occur in those equilibria. Elimination algorithms are employed to reduce systems of polynomial equations, which allows for equilibria to be visualized, and also to demonstrate which models within the parameter space exhibit bifurcation. We find the `DixonEDF` algorithm for the Dixon resultant[2], written in the computer algebra system *Fermat*, to be capable to complete the computation for the challenging system of equations that represents bifurcation, while attempts with other algorithms were terminated after several hours.

Keywords
Double Pendulum, Bifurcation, Polynomial System Solving, Elimination, Dixon Resultant

References

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